ETSI de CAMINOS, CANALES Y PUERTOS DE MADRID

METODOS MATEMATICOS DE LAS TECNICAS (Matemáticas) – 2004-05

FORMULARIO PRIMER PARCIAL

No está permitido añadir otras informaciones a este formulario

1. Algebra tensorial

Fórmula de expulsión

$$\vec{a}\times(\vec{b}\times\vec{c}\,) = \vec{b}\,(\vec{a}\cdot\vec{c}\,) - \vec{c}\ \ (\vec{a}\cdot\vec{b}\,)$$

Representación de una Rotación;

$$\mathbf{R} = \cos\theta\,\mathbf{1} + (1-\cos\theta)\vec{e}\otimes\vec{e} + \operatorname{sen}\theta\,\vec{e}\times$$

Representación de una simetría o reflexión

$$\mathbf{H} = \mathbf{1} - 2\,\vec{e}\otimes\vec{e}$$

2. Coordenadas curvilíneas

a) *Cambio de sistema de coordenadas*: Dadas las relaciones entre las variables curvilíneas:

$$x^{\mathrm{i}} = x^{\mathrm{i}}(\hat{x}^1, \hat{x}^2, \hat{x}^3)$$

entonces la matriz del cambio de base es la matriz jacobiana F que verifica:

$$F = \left[\frac{\partial(x^1, x^2, x^3)}{\partial(\hat{x}^1, \hat{x}^2, \hat{x}^3)}\right] = \begin{bmatrix} | & | & | \\ \underline{g}_1 & \underline{g}_2 & \underline{g}_3 \\ | & | & | \end{bmatrix}_{\text{en base } \{\vec{g}_{\mathrm{i}}\}}$$

y si $\vec{u}$ es un campo dado por $\vec{u} = u^{\mathrm{i}}\ \underline{g}_{\mathrm{i}} = u_{\mathrm{i}}\,\underline{g}^{\mathrm{i}}$, entonces:

$$\begin{pmatrix} \hat{u}^1 \\ \hat{u}^2 \\ \hat{u}^3 \end{pmatrix} = F^{-1}\cdot\begin{pmatrix} u^1 \\ u^2 \\ u^3 \end{pmatrix} \ ; \ \begin{pmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \hat{u}_3 \end{pmatrix} = F^{t}\cdot\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

y si $\mathbf{T}$ es un campo tensorial dado por $\mathbf{T} = t^{\mathrm{i}}_{\ \mathrm{j}}\ \underline{g}_{\mathrm{i}}\otimes\underline{g}^{\mathrm{j}} = \hat{t}^{\mathrm{p}}_{\ \mathrm{q}}\ \hat{\underline{g}}_{\mathrm{p}}\otimes\hat{\underline{g}}^{\mathrm{q}}$, etc.., entonces:

$$\left[\hat{t}^{\mathrm{p}}_{\ \mathrm{q}}\right] = F^{-1}\cdot\left[t^{\mathrm{i}}_{\ \mathrm{j}}\right]\cdot F \ ; \ \left[\hat{t}_{\mathrm{p}}^{\ \mathrm{q}}\right] = F^{\mathrm{t}}\cdot\left[t_{\mathrm{i}}^{\ \mathrm{j}}\right]\cdot F^{-\mathrm{t}}$$

$$\left[\hat{t}_{\mathrm{ij}}\right] = F^{\mathrm{t}}\cdot\left[t_{\mathrm{pq}}\right]\cdot F \ ; \ \left[\hat{t}^{\mathrm{ij}}\right] = F^{-1}\cdot\left[t^{\mathrm{pq}}\right]\cdot F^{-\mathrm{t}} \quad \underline{g}\ \hat{\underline{g}}$$

b) *Símbolos de Chistoffel de esféricas*:

$$\left[\Gamma^{\mathrm{k}}_{\mathrm{ij}}\right]_{\mathrm{j}=m,\mathrm{k}=f,\mathrm{i}=c} = \left[\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{r} & 0 \\ 0 & 0 & \frac{1}{r} \end{bmatrix} \begin{bmatrix} 0 & -r & 0 \\ \frac{1}{r} & 0 & 0 \\ 0 & 0 & \cot\varphi \end{bmatrix} \begin{bmatrix} 0 & 0 & -r\operatorname{sen}^2\varphi \\ 0 & 0 & -\operatorname{sen}\varphi\cos\varphi \\ \frac{1}{r} & \cot\varphi & 0 \end{bmatrix}\right]$$

3. Diferenciación de campos

a) *Componentes de las derivadas covariantes*:

$$u^{i}{}_{,j} = \frac{\partial u^{i}}{\partial x^{j}} + u^{h}\Gamma^{i}{}_{hj} \qquad u_{i,j} = \frac{\partial u_{i}}{\partial x^{j}} - u_{h}\Gamma^{h}{}_{ij}$$

$$t^{ij}{}_{,k} = \frac{\partial t^{ij}}{\partial x^{k}} + t^{hj}\Gamma^{i}{}_{hk} + t^{ih}\Gamma^{j}{}_{hk} \qquad t_{ij,k} = \frac{\partial t_{ij}}{\partial x^{k}} - t_{hj}\Gamma^{h}{}_{ik} - t_{ih}\Gamma^{h}{}_{jk}$$

$$t^{i}{}_{j,k} = \frac{\partial t^{i}{}_{j}}{\partial x^{k}} + t^{h}{}_{j}\Gamma^{i}{}_{hk} - t^{i}{}_{h}\Gamma^{h}{}_{jk} \qquad t_{i}{}^{j}{}_{,k} = \frac{\partial t_{i}{}^{j}}{\partial x^{k}} - t_{h}{}^{j}\Gamma^{h}{}_{ik} + t_{i}{}^{h}\Gamma^{j}{}_{hk}$$

b) *Reglas de actuación de los operadores diferenciales* frente a productos de campos:

$$\underline{\nabla}(fg) = (\underline{\nabla} f)g + f(\underline{\nabla} g)$$
$$\underline{\nabla}\cdot(f\underline{u}) = (\underline{\nabla} f)\cdot\underline{u} + f\,\underline{\nabla}\cdot\underline{u}$$
$$\underline{\nabla}\times(f\underline{u}) = \underline{\nabla} f\times\underline{u} + f\,\underline{\nabla}\times\underline{u}$$
$$\underline{\nabla}(\underline{u}\cdot\underline{v}) = \underline{v}\cdot\underline{\nabla}\otimes\underline{u} + \underline{u}\cdot\underline{\nabla}\otimes\underline{v} + \underline{u}\times(\underline{\nabla}\times\underline{v}) + \underline{v}\times(\underline{\nabla}\times\underline{u})$$
$$\underline{\nabla}\cdot(\underline{u}\times\underline{v}) = \underline{v}\cdot\underline{\nabla}\times\underline{u} - \underline{u}\cdot\underline{\nabla}\times\underline{v}$$
$$\underline{\nabla}\times(\underline{u}\times\underline{v}) = \underline{v}\cdot\underline{\nabla}\otimes\underline{u} - \underline{u}\cdot\underline{\nabla}\otimes\underline{v} - (\underline{\nabla}\cdot\underline{u})\underline{v} + (\underline{\nabla}\cdot\underline{v})\underline{u}$$
$$\underline{\nabla}\cdot(\underline{u}\otimes\underline{v}) = (\underline{\nabla}\cdot\underline{u})\underline{v} + \underline{u}\cdot\underline{\nabla}\otimes\underline{v}$$
$$\underline{\nabla}\times(\underline{\nabla}\times\underline{u}) = (\underline{\nabla}\otimes\underline{\nabla})\underline{u} - \underline{\nabla}^{2}\underline{u}$$

c) *Operadores diferenciales en coordenadas curvilíneas* (x^1,x^2,x^3):

$$\underline{\nabla} f = \frac{\partial f}{\partial x^{i}}\,\underline{g}^{i}$$

$$\underline{\nabla}\cdot\underline{u} = u^{i}{}_{,i} = g^{ij}u_{j,i} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{i}}\left[\sqrt{g}\,u^{i}\right]$$

$$\underline{\nabla}\times\underline{u} = e^{ijk}u_{i,j}\,\underline{g}_{k} = \frac{1}{\sqrt{g}}\begin{vmatrix} \underline{g}_{1} & \underline{g}_{2} & \underline{g}_{3} \\ \dfrac{\partial}{\partial x^{1}} & \dfrac{\partial}{\partial x^{2}} & \dfrac{\partial}{\partial x^{3}} \\ u_{1} & u_{2} & u_{3} \end{vmatrix}$$

$$\underline{\nabla}\underline{u} = \underline{\nabla}\otimes\underline{u} = u^{i}{}_{,j}\,\underline{g}^{j}\otimes\underline{g}_{i} = u_{i,j}\,\underline{g}^{j}\otimes\underline{g}^{i}$$

$$\underline{\nabla}^{2} f = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^{i}}\left[\sqrt{g}\,g^{ij}\frac{\partial f}{\partial x^{j}}\right]$$